CALCULATING THE COOLING TIME OF SOLID BODIES OF VARIOUS GEOMETRIC SHAPES

A. G. Fikiin and I. K. Fikiina

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A simple analytic expression is obtained for determination of the cooling time of solid bodies, in particular, foodstuffs of various geometric shapes.

Employing the results presented in [4], [1, 2] have explained the change in the temperature field occurring in the cooling of foodstuffs. The goal of the present study is the formulation of an analytic expression of the functional dependence between the criteria Bi and Fo and the parameters of the temperature field θ upon cooling of solid objects (foodstuffs), and finally, the determination of a simple mathematical formula for calculating the duration of the process.

In [3] such a formula was presented for the cooling of objects of laminar form. The results of studies on objects of cylindrical and spherical shapes are presented below, and the analytic expression is generalized for objects of various shapes.

In solution of these problems, the well-known solutions in the form of convergent infinite series, [1-4] were employed:

for cylindrical objects

$$\theta = 2\sum_{i=1}^{\infty}\frac{I_1(\mu_i)}{\mu_i\left[I_0^2(\mu_i)+I_1^2(\mu_i)\right]}\exp(-\mu_i^2\,\mathrm{Fo})\,I_0\left(\mu_i\frac{X}{R}\right), \tag{1}$$

for spherical objects

$$\theta = 2\sum_{i=1}^{\infty}\frac{\sin\mu_i-\mu_i\cos\mu_i}{\mu_i-\sin\mu_i\cos\mu_i}\exp(-\mu_i^2\,\mathrm{Fo})\,\frac{\sin\left(\mu_i\frac{X}{R}\right)}{\mu_i\frac{X}{R}}. \tag{2}$$

In the technological process, it is the temperature at the center of the product, i.e., at X = 0, which is controlled. Hence, Eqs. (1, 2) take on the forms:

for cylinders

$$\theta = 2\sum_{i=1}^{\infty}\frac{I_1(\mu_i)}{\mu_i\left[I_0^2(\mu_i)+I_1^2(\mu_i)\right]}\exp(-\mu_i^2\,\mathrm{Fo}), \tag{3}$$

for spheres

$$\theta = 2\sum_{i=1}^{\infty}\frac{\sin\mu_i-\mu_i\cos\mu_i}{\mu_i-\sin\mu_i\cos\mu_i}\exp(-\mu_i^2\,\mathrm{Fo}), \tag{4}$$

where μ_i are the roots of the characteristic equation $\mu I_1(\mu) = \mathrm{Bi} I_0(\mu)$; I_0, I_1 are Bessel functions of the first

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type, of zeroth and first order. The values of μ_i (for i = 1-6) for various Bi (from 0 to ∞) are given in tabular form in [4].

Thus it follows that for Eqs. (3, 4) one may write a generalized dependence of the form

$$\theta = f(\mathrm{Bi}, \mathrm{Fo}). \tag{5}$$

After calculations of series (3) and (4) and analysis of the parameter values in Eq. (5), it was established that the functional dependence between Bi, Fo, and θ has the form

$$\mathrm{Fo} = k \lg \theta + n, \tag{6}$$

where k = f(Bi).

Values were determined for Bi from (0.01 to ∞), Fo from (0 to 400), and θ (from 0.001 to 1). The tensioned filament method [1, 2] was used. The following values for n were obtained: for a cylinder, 0.06; for a sphere, 0.04.

Regarding k as a function of Bi, we obtain the following analytic relationship:

$$k = \frac{c}{\mathrm{Bi}} + d.$$

The coefficients c and d were determined by the method of mean deviations [1, 2], using the system

$$c \sum_{i=1}^{m} f(X_i) + md = \sum_{i=1}^{m} y_i,$$

$$c \sum_{i=m\pm 1}^{n} f(X_i) + (n-m) d = \sum_{i=m\pm 1}^{n} y_i,$$

where m indicates the number of observations in the first group, which was chosen such that it be equal to n, if n is even, and to n ± 1, if n is odd. Values obtained for a cylinder were c = 1.15, d = 0.4; for a sphere, c = 0.767, d = 0.27.

Substituting the k values obtained in Eq. (6), we obtain the following expression for the functional relationship between Fo, Bi, and θ:

for a cylinder

$$\mathrm{Fo} = -\left(\frac{1.15}{\mathrm{Bi}} + 0{,}4\right) \lg \theta + 0.06, \tag{7}$$

for a sphere

$$\mathrm{Fo} = -\left(\frac{0.767}{\mathrm{Bi}} + 0.27\right) \lg \theta + 0.04. \tag{8}$$

As is evident, there are similarities between Eqs. (7) and (8). One might add to them Eq. (4), for laminar objects, obtained in [3]. As a result, one can write a general mathematical equation for determination of the duration of cooling time of foodstuffs of various geometric shapes:

$$\tau = -A \frac{R^2}{a} \left[\left(\frac{2.3}{\mathrm{Bi}} + 0.8 \right) \lg \theta + 0.12 \right], \tag{9}$$

where

$$A = \begin{cases} 1 & \text{for a lamina;} \\ 1/2 & \text{for a cylinder;} \\ 1/3 & \text{for a sphere.} \end{cases}$$

Comparing data obtained with the aid of Eq. (9) with those from Eqs. (2) and (3) and Eq. (2) from [3], very good agreement was noted. Moreover, the results agree quite well with data obtained by actual cooling of products having the corresponding geometric forms.

Discrepancies were noted only at high values of Bi and Fo.

NOTATION

$\theta = (t_\tau - t_0)/(t_H - t_0)$	is the temperature field;
t_τ	is the temperature at center of product at end of process (after time τ);
t_H	is the initial product temperature;
t_0	is the coolant medium temperature;
R	is the distance from product surface to center, i.e., cylinder or sphere radius;
a	is the thermal conductivity;
$a = \lambda/c\gamma$;	
$Bi = (\alpha/\lambda)R$	is the Biot number;
$Fo = a\tau/R^2$	is the Fourier number.

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